**Modelling with MATLAB Assignment 5 2021: Getting to grips with current research**

This assignment involves the research paper “Dispersal-induced instability in complex ecosystems” by Baron and Galla, published in *Nature Communications* in 2020. The paper and its Supplementary Information are provided for you on Moodle. The original Levin-Segel paper is also provided for completeness.

**You do not need to understand these papers in their entirety in order to answer the questions in this assignment. Your task is to implement, to test, and possibly to extend, some of the mathematical ideas, and to understand some of the strengths and limitations of the modelling involved.**

You must **not** contact the authors in relation to this assignment until the assignment deadline has passed. To do so might constitute academic misconduct. The authors have agreed not to answer any such contact, and to refer any inappropriate communications to the University of York.

Your solutions should be uploaded to Moodle as a single written document, which may contain graphics and mathematics (but which does not just list your code), and which makes clear links to well-labelled MATLAB files which should be uploaded to Moodle (as .m files) at the same time. Your solutions may be in PDF (e.g. generated via LaTeX), Word, or any other appropriate format, but they must NOT depend on the marker having access to any additional software beyond a PDF reader, Microsoft Word, and a copy of MATLAB. Credit will be given for providing working code, and for providing suitable comments within the code to allow it to be used accurately. **Simply submitting a collection of MATLAB files is not enough.**

There are 5 questions in this assignment. Each question carries 20 marks.

**Question 5 is different for H-level and M-level students. Make sure you answer the question appropriate to your module code.**

Your answers need to be uploaded on Moodle by **1800 on MONDAY** **10th JANUARY 2021**, please.

In Q1-Q3, you will explore the role of the interaction matrix **A** in Baron and Galla (2020). Explicitly, you will ask whether the results of Baron and Galla (2020) apply for a random interaction matrix **A** (in the sense of May’s original papers in the early 1970s, studied in practicals) rather than the predator-prey structured matrix used in Baron and Galla (2020).

**Q1.** By separately defining matrices **D**, **d** and **A** and then combining them appropriately, write MATLAB code to generate random matrices of the form described in equation (2) and Figure 1 of Baron and Galla (2020) under the following assumptions:

Assume the number of species N is even, and that the first N/2 species diffuse as “prey” and the remaining D/2 species diffuse as “predators”.

Assume the parameter values are Du = 1, Dv = 5, du = dv = 1.

Assume that the interaction matrix **A** is a fully connected square matrix of independent Gaussian random variables with mean 0 and standard deviation σ (i.e. there is no trophic structure, unlike the predator-prey **A** matrix defined in Baron and Galla (2020)).

Give an example of matrices **D**, **d**, **A** and **M**q for the case where N = 8, q = 1 and σ = 0.1.

**Q2.** Plot the eigenvalue spectra of random matrices from Q1 above, for σ = 0.1, N = 100, for the three cases q = 0, q = 0.2 and q = 0.5, and comment on the stability of the underlying dynamical system in each case.

**Q3.** Produce a version of Baron and Galla’s Figure 2d, but using the matrices defined in Q2 (above) for values of q between 0 and 1, and for some suitable value (or values) of σ. Hence, or otherwise, state whether you agree with the assertion,

*“trophic structure is necessary for dispersal-induced instability in complex ecosystems.”*

Q4 and Q5 refer to a non-spatial version of the Levin-Segel model used in Baron and Galla (2020); the answers to these questions should be independent of your answers to Q1-Q3.

Equations (11) in Baron and Galla (2020) define the Levin-Segel dynamical system for the populations of prey species ui and predator species vj. In the simplest case with one prey species u and one predator species v, and in the absence of spatial structure, the dynamical system can be written

u’ = a u + e u2 - b u v

v’ = c u v – d v2

where the parameters a, b, c d and e are positive constants and the ’ denotes a derivative with respect to time. You can assume that the dynamic variables u and v, and the parameters a, b, c d and e are written in a dimensionless form i.e. you do not need to worry about units.

**Q4.** Write a MATLAB script to evaluate the temporal derivatives for the Levin-Segel system defined above, with parameter values a = b = c = d = 1 and e = 0.1.

Use this function, together with other MATLAB functions as appropriate, to show that the point (1.111 , 1.111) (to 3 decimal places) is a stable fixed point for this system.

**Q5. H-LEVEL ONLY**

Suppose that the value of e is a random variable distributed uniformly on the interval [0, 0.5]. The other parameter values are as in question 4.

Use a Monte Carlo method to assess the probability that the Levin-Segel system, starting at an initial value of [1, 1], reaches a prey or predator population greater than 2 in the first 20 time units.

**Q5. M-LEVEL ONLY**

Suppose that, for the system described in Q4 (above), there is a delay between the predation event and the subsequent increase in predator population.

Show how such a delay could be incorporated into the dynamical system, stating clearly any assumptions you make. For your new delay-differential equations, and for the parameter values in Q4 (above), use MATLAB to assess whether the addition of a delay can cause instability in your system.